Cosmology Exam

30 October 2015, 14.00-17.00 uur

Rules:

- Closed notes part $\leq$ 2 hours, open notes part all remaining time.
- Please formulate clearly and concisely. Read carefully.
- Grade $= 1 + \frac{score}{10}$.

1. **General Relativity [5+5=10 points]**

   (a) Reason from the weak equivalence principle to explain what the most fundamental property of the Einstein equation is (use less than 200 words).

   (b) State the connection between de Sitter space and black holes, and argue thermodynamically that an exponentially expanding universe must enjoy quantum fluctuations (use less than 200 words).

2. **Geometry of the Universe [5+5=10 points]**

   (a) State what principle underlies the Robertson-Walker metric and explain it (use less than 200 words).

   (b) Argue whether the Anthropic principle is classical, quantum-mechanical or both (use less than 300 words).

3. **Big Bang Nucleosynthesis [5+5=10 points]**

   (a) Explain how neutrons survive the hot big bang, even though free neutrons decay in 15 minutes (use less than 200 words).

   (b) Explain the significance of the $\sim$ 1 MeV energy scale, about 10^{10} K, in the early universe (use less than 200 words).

4. **Expansion of the Universe [5+5=10 points]**

   (a) Give the overall dependence on scale factor a of a radiation, matter and vacuum energy dominated universe. Provide physical arguments (use less than 100 words for each scaling law).

   (b) Explain the geometric and particle physics meanings of the cosmological constant in the Einstein equation (use less than 300 words).

5. **The CMB [5+5=10 points]**

   (a) Explain what the last scattering surface is at a redshift of about 10^3 (use less than 200 words).

   (b) Relate the temperature of the CMB to Olbers' paradox and its resolution (use less than 200 words).

> End of closed notes part, hand in your work and p.t.o.

6. **Cosmic Inflation [10 points]**

Write down the Friedmann equation for a vacuum energy dominated universe. Solve the equation. Discuss the solution in relation to the flatness and horizon problem.

7. **Sound Propagation [10 points]**

Write down the Euler equation for a relativistic fluid that obeys a perfect gas law. Show that the sound speed is $c/\sqrt{3}$ and discuss what this means for density perturbations and the CMB.

8. **Expansion and Horizons [10 points]**

Compute the particle horizon, using conformal time for photons, for a radiation dominated, a matter dominated and a vacuum energy dominated universe. Highlight one significant difference and one significant similarity among these three cosmologies.

9. **Expansion and Redshift [10 points]**

Consider the cosmic expansion of a photon fluid. Derive the temperature dependence of the CMB with redshift. Furthermore, argue why neutrinos mimic the behavior of photons even though they are fermions with a small mass.

End of exam, hand in your work.